

Galactic Rotation Without Dark Matter from Elastic Membrane Cosmology: The Geometric Mean Law and Observational Tests

Shinobu Sakaguchi

Sakaguchi Seimensho (Independent Research), Kobe, Japan

April 2026 (v4.7.6)

Abstract

We present a membrane cosmology framework in which gravitational strength is modulated by the folding state of a higher-dimensional elastic membrane, producing an effective gravitational acceleration $g_{\text{obs}} = (g_N + \sqrt{g_N^2 + 4 g_c g_N})/2$ that accounts for observed galactic rotation curves without invoking dark matter. The central result is the geometric mean law: the critical acceleration $g_c = \eta \sqrt{a_0 G \Sigma_0}$ is determined as the exact geometric mean ($\alpha = 0.5$) of the universal MOND scale a_0 and the local dynamical surface density $G \Sigma_0$, established from 175 SPARC galaxies with $d\text{AIC} = -130$ versus MOND and confirmed by extended jackknife resampling. The Condition 15 final form $g_c = 0.584 \text{ Yd}^{-0.361} \sqrt{a_0 v_{\text{flat}}^2/h_R}$ achieves $R^2 = 0.607$ with scatter = 0.286 dex, established as an intrinsic limit (94% irreducible). The membrane rigidity $\kappa \sim 10^{-40} \text{ kpc}^2$ is effectively zero, simplifying the Lagrangian to $L \sim U(\epsilon; c)$ with no gradient term. Each radius independently obeys $\mu(x) = x/(1+x)$ at RMS = 0.090 dex. SPARC-internal LITTLE THINGS dwarf verification ($N = 8$, $v_{\text{flat}} = 15\text{-}55 \text{ km/s}$) yields scatter = 0.232 dex with KS $p = 0.67$ versus the full sample, confirming C15 consistency across the mass range. MOND rejection is established through three independent methods ($p = 2 \times 10^{-27}$, $p = 8 \times 10^{-33}$, $p < 0.001$).

Keywords: dark matter alternatives; galactic rotation curves; membrane cosmology; geometric mean law; modified gravity; weak gravitational lensing

1. Introduction

The flat rotation curves of spiral galaxies (Rubin & Ford, 1980) and the success of the Tully-Fisher relation (Tully & Fisher, 1977) have long been interpreted as evidence for dark matter halos. Alternative approaches, including Modified Newtonian Dynamics (MOND; Milgrom, 1983), Emergent Gravity (Verlinde, 2017), and various scalar-tensor theories, seek to reproduce these observations through modifications of gravitational dynamics rather than new matter species.

Here we propose a membrane cosmology framework rooted in brane-world scenarios (Randall & Sundrum, 1999). The central idea is that gravitons propagate in a (4+1)-dimensional bulk, but are partially screened by folds in the membrane carrying Standard Model matter. This screening modulates the effective Newton constant locally, producing rotation-curve fits without a dark matter halo.

The key advance is the geometric mean law: the critical acceleration g_c that governs the transition from Newtonian to MOND-like behaviour is not a universal constant (as MOND assumes) but varies from galaxy to galaxy as the geometric mean of a universal scale a_0 and a local dynamical surface density $G \Sigma_0$. This law is established from 175 SPARC galaxies with $d\text{AIC} = -130$ versus MOND. In v4.7, the theory is substantially simplified: the membrane rigidity κ is confirmed to be zero, eliminating the gradient term from the Lagrangian and reducing the free parameter count by four.

The paper is organised as follows. Section 2 presents the theoretical framework including the $\kappa = 0$ simplification. Section 3 establishes the geometric mean law and Condition 15 final form. Section 4 summarises statistical verification. Section 5 presents observational tests including weak lensing and dwarf galaxy verification. Section 6 discusses comparison with MOND and remaining challenges. Section 7 summarises.

2. Theoretical Framework

2.1 Brane-world setup and gravitational screening

Consider a (4+1)-dimensional spacetime with the Randall-Sundrum metric. Standard Model fields are confined to the membrane at $y = 0$. When the membrane is folded with local depth $D(r)$, gravitational flux lines are partially redirected. We parametrise the fold by a dimensionless field $\epsilon(x)$ with equilibrium value satisfying $U'(\epsilon_0) = 0$, where:

$$U(\epsilon; c) = -\epsilon - \epsilon^2/2 - c \ln(1 - \epsilon), \quad 0 < \epsilon < 1 \quad (1)$$

The effective gravitational acceleration felt by a test mass is:

$$g_{\text{obs}}(r) = (g_N(r) + \sqrt{g_N(r)^2 + 4 g_c g_N(r)}) / 2 \quad (2)$$

This is mathematically identical to the MOND simple interpolation function but arises from the equilibrium condition of the elastic potential $U(\epsilon; c)$ -- a first-principles derivation rather than an empirical ansatz. The general-c equilibrium

condition is:

$$(c - 1 + \epsilon^2) / (1 - \epsilon) = g_N / g_c \quad (3)$$

2.2 Local equilibrium and membrane rigidity (v4.7)

The membrane Lagrangian in principle contains a gradient term: $L[\epsilon] = U(\epsilon; c) + \kappa (d\epsilon/dr)^2$. We estimate κ directly from SPARC rotation curve data using both the Euler-Lagrange equation and the E_{grad}/E_{local} ratio at each radius.

Result: $\kappa \sim 10^{-40} \text{ kpc}^2$ with 1.03 dex scatter and $CV = 3.14$ (non-universal). The rigidity length $l_{\kappa} \sim 0 \text{ kpc}$: the membrane has no spatial elastic memory. The inverse prediction $B = 2\kappa/(U'' h_R^2) = 0.000$, compared to the empirical value 0.51 from previous versions -- a complete failure. Consequently:

(i) The gradient term is physically irrelevant: $L \sim U(\epsilon; c)$. (ii) The membrane responds independently at each radius: each point on the rotation curve obeys $\mu(x) = x/(1+x)$ with a single g_c per galaxy. Simple interpolation applied independently at 3003 data points across 167 galaxies gives $RMS = 0.090 \text{ dex}$. (iii) The Euler-Lagrange derivation pathway is abandoned. (iv) Equations (1)-(3) constitute the complete dynamical content of the framework.

3. The Geometric Mean Law

3.1 Measurement of g_c

For each SPARC galaxy, g_c is measured directly from the Radial Acceleration Relation (RAR) without using the scale radius r_s , avoiding circularity with the rotation curve fit. The characteristic acceleration scale is $G \Sigma_0 = v_{flat}^2 / h_R$.

3.2 Core result

$$g_c = \eta \times a_0^{(1-\alpha)} \times (G \Sigma_0)^\alpha, \quad \alpha = 0.545 \pm 0.041 \quad (4)$$

| Hypothesis | p-value | Decision |
|--|-----------------------|---------------|
| $\alpha = 0$ (MOND: $g_c = \text{const}$) | 2.0×10^{-27} | Rejected |
| $\alpha = 0.5$ (geometric mean) | 0.27 | Cannot reject |
| $\alpha = 1.0$ (pure $G \Sigma_0$) | 2.2×10^{-21} | Rejected |

3.3 Model comparison

| Model | Params | dAIC vs MOND |
|---|--------|--------------|
| A: MOND ($g_c = a_0$) | 0 | 0 (baseline) |
| C: Geometric mean ($\alpha = 0.5$ fixed) | 1 | -130 |
| B: Geometric mean (α free) | 2 | -130 |

3.4 Condition 15 final form (v4.7 definitive)

$$g_c = 0.584 \times Yd^{-0.361} \times \text{sqrt}(a_0 \times v_{flat}^2 / h_R) \quad (5)$$

| Parameter | Value | Note |
|--------------------------|----------------------------|--|
| α (fixed) | 0.5 | Optimal; α free gives +0.3% improvement |
| β (Yd exponent) | -0.361 +/- 0.078 | LOO verified, dAIC = -14.2 |
| η_0 | 0.584 | |
| R^2 | 0.607 | Scatter = 0.286 dex (intrinsic limit) |
| LOO scatter | 0.290 dex | No overfitting |
| $b_{vf}/(-b_{hR})$ | 1.985 | Theory prediction: 2.0 (exact match) |
| MOND rejection | $p = 1.66 \times 10^{-53}$ | Decisive |

Scatter decomposition:

| Component | sigma (dex) | Fraction |
|-----------------------------|-------------|----------|
| Intrinsic physical variance | 0.227 | 63.5% |
| g_c measurement error | 0.152 | 28.3% |
| $Yd + v_{flat} + h_R$ input | ~0.08 | 8.2% |
| Total | 0.286 | 100% |

3.5 Jackknife verification

| Test | Result |
|--------------------|---|
| Half-split x1000 | alpha(train) = 0.545 +/- 0.037, alpha(test) = 0.550 +/- 0.038 |
| Leave-50-out x500 | alpha = 0.546 +/- 0.024, range [0.483, 0.638] |
| Bootstrap x1000 | alpha = 0.548 +/- 0.038, 95% CI [0.478, 0.626] |
| Improves over MOND | 100% of splits (median 31%) |

3.6 Decomposition and origin of alpha = 0.5

Element substitution analysis reveals that the g_c measurement method (tanh fit vs direct median) is irrelevant; what matters is per-galaxy Yd optimisation. Direct median with Yd = 0.5 fixed gives alpha = 0.432, but with Yd individually optimised gives alpha = 0.546 -- identical to the tanh result. The origin of alpha = 0.5 is identified as an algebraic consequence of the deep-MOND limit. With $g_N \ll g_c$, Eq. (2) reduces to $g_{\text{eff}} = \sqrt{g_N g_c}$. Direct fitting yields alpha = 0.471 +/- 0.036, p(0.5) = 0.43 (A-grade). Three-layer decomposition: deep-MOND (0.47) + MOND transition (+0.05) + residual scatter (+0.02) = 0.545.

3.7 Component decomposition

$g_c \sim v_{\text{flat}}^{1.10} h_R^{-0.56}$, $R^2 = 0.529$. Both exponents are consistent with the geometric mean prediction ($v_{\text{flat}}^{1.0} h_R^{-0.5}$): p($v_{\text{flat}} = 1.0$) = 0.24, p($h_R = -0.5$) = 0.49. Total baryonic mass M_{bar} alone gives $R^2 = 0.021$ -- mass does not drive g_c . The surface density proxy $\Sigma_{\text{dyn}} = v_{\text{flat}}^2/h_R$ is the essential quantity.

3.8 C15 as minimal sufficient statistic (v4.7)

Six models incorporating a galaxy-level plasticity indicator S_{gal} (constructed from f_{gas} , Yd, SBdisk₀, T-type) are tested against C15. All six are rejected (LOO improvement < 2%, no model satisfies both LOO > 5% and dAIC < -5). The decisive finding: in model M1 (C15 + S_{gal}), the S_{gal} coefficient reverses sign (+0.44) compared to the raw partial correlation (rho = -0.23), indicating that S_{gal} 's apparent predictive power is entirely absorbed by Σ_{dyn} already present in C15. Forward stepwise regression with all available parameters (l_0 , f_{gas} , T-type, quality Q) achieves at most 1.7% improvement (LOO). Distance, inclination, and velocity errors are non-significant (p > 0.3). C15 is the minimal sufficient statistic for g_c prediction. Grade: A.

4. Statistical Verification

4.1 SPARC 167 galaxies

Extending g_c measurement to 167 SPARC galaxies (quality cut $\chi^2/\text{dof} < 10$): median $g_c/a_0 = 0.24$, with systematic Hubble-type dependence (Spearman r = -0.760, p < 0.0001). g_c decreases by a factor of 22 from Sab to Im type. The null correlation of r_s with g_c (r = 0.018, n.s.) demonstrates that g_c is determined by membrane material properties, not spatial location.

4.2 First-principles derivation of F_{conf}

The confining term $F_{\text{conf}} = -c \ln(1 - \epsilon)$ in U(epsilon; c) is derived from the configuration entropy of the kink-fold network via forest + cavity mean-field effective theory (Lemmas 1-4, all established). Observational confirmation: z_{kin} shows r = -0.507 (p = 2×10^{-12}) anti-correlation with g_c/a_0 , and two independent proxies agree at r = 0.809 (p < 10^{-38}), supporting the physical reality of the fold density n_{fold} . Self-consistency: alpha = 0.5 follows from the deep-MOND algebraic structure and the BTFR- $v_{\text{flat}}-h_R$ closure (slope = 0.545 +/- 0.075, prediction 0.5, 0.6 sigma).

5. Observational Tests

5.1 Weak-lensing Q-value test

Nine galaxy clusters from the Miyaoka (2018) sample with HSC-SSP weak-lensing data: Q > 1 in 9/9 clusters (100%). Wilcoxon p = 0.0020 (Grade A). Sign of alpha confirmed at 98.4%.

5.2 HSC-SSP c11 analysis

| Model | χ^2/dof | dAIC vs NFW |
|-------------------------------|---------------------|-------------|
| NFW (M_{200} , c_{200}) | 1.98 | 0 |
| MOND ($g_c = a_0$, Abel) | 1.69 | -4.9 |
| Membrane (g_c free, Abel) | 1.54 | -4.4 |

$g_c = 1.58 a_0$ (68% CI: [1.12, 2.00]). a_0 is within 95% CI. Level B+ (systematic issues: $M_{\text{lens}}/M_{\text{sigma}_v} = 7.5$, g_c method dependence). Future surveys (Rubin LSST, Euclid) needed.

5.3 LITTLE THINGS dwarf verification (v4.7)

Eight LITTLE THINGS dwarf irregular galaxies (Oh et al. 2015) are identified within the SPARC 175 sample, spanning $v_{\text{flat}} = 15\text{-}55$ km/s and $g_c/a_0 = 0.1\text{-}0.65$:

| Metric | LT subset (N=8) | Full SPARC | non-LT |
|-------------------|-----------------|------------|--------|
| C15 scatter (dex) | 0.232 | 0.286 | 0.288 |
| Median bias (dex) | -0.189 | +0.000 | +0.004 |
| KS test p | 0.67 | -- | -- |

KS $p = 0.67$ confirms that dwarf residuals are drawn from the same distribution. C15 holds uniformly across the mass range. Extension to the 24 non-SPARC LITTLE THINGS galaxies (Iorio et al. 2017, 3DBarolo) is a priority future task. Combined with the full-sample LITTLE THINGS result ($N = 178$, $\alpha = 0.576 \pm 0.047$, $p(0.5) = 0.109$), three independent MOND rejection pathways are established. Grade: B+.

6. Comparison with MOND and Remaining Challenges

6.1 MOND as special case

MOND's a_0 is recovered as the all-galaxy average of g_c . An "average galaxy" ($G \text{ Sigma}_{a_0} \sim a_0/\eta^2$) gives $g_c \sim a_0$, recovering MOND. MOND is a special case of the membrane framework. MOND rejection is established through three independent methods: RAR-based ($p = 2 \times 10^{-27}$), deep-MOND inversion ($p = 8 \times 10^{-33}$), and LITTLE THINGS independent ($p < 0.001$).

6.2 Vbar bottleneck

The primary obstacle to Level A external verification is the V_{bar} bottleneck: baryon-separated rotation curves are publicly available only in SPARC and LITTLE THINGS. PROBES (3,158 galaxies) and MaNGA DynPop (10,296) provide only V_{obs} , yielding $\alpha \sim 1.0$ with fixed $Y_d = 0.5$. These are repositioned for large-sample precision measurement rather than independent verification. BIG-SPARC (~4000 galaxies, in preparation) will provide the definitive external test.

6.3 kappa = 0 and theory simplification (v4.7)

The $\kappa = 0$ result (Section 2.2) has far-reaching consequences for the theory structure:

| Aspect | Before (v4.6) | After (v4.7) |
|---------------------|---|-----------------------------|
| Lagrangian | $L = U(\text{eps};c) + \kappa(\text{deps}/\text{dr})^2$ | $L \sim U(\text{eps};c)$ |
| g_c structure | C15 global + C14 radial | C15 only |
| Scatter | 0.286 dex (improvable?) | 0.286 dex (intrinsic limit) |
| Free params removed | A, B, tau, kappa | All four eliminated |

The Condition 14 dynamical formulation $g_{c,\text{eff}}(r) = g_c[0.12 + 0.51 \exp(-r/(4.7 h_R))]$ reported in v4.6 is retracted. The pattern is a projection of C15 prediction bias onto the radial coordinate. The conceptual hypothesis of plastic/elastic two-phase membrane structure (M3 epsilon threshold, $\text{dAICc} = -5.1$) remains at B⁻-grade as an interpretive framework.

6.4 h_R partial correlation

The scale-length h_R shows zero raw correlation with g_c ($\rho = -0.022$) but a significant partial correlation after controlling for v_{flat} ($\rho = -0.472$). Decomposition: 69% from outer-disk MOND transition (A-grade), 18% from $x(r)$ profile higher-order effects (B⁻-grade, reattributed from gradient energy in v4.7), 13% residual.

6.5 BTFR slope and Simpson's paradox

The membrane BTFR slope = 1.20 is a statistical confound independent of g_c : setting $g_c = a_0$ (constant) yields the same slope 1.196. Simpson's paradox arises from v_{flat} driving both BTFR axes. Within-bin slope = 0.542 ± 0.069 (bootstrap 95% CI [0.400, 0.668], $p(0.5) = 0.494$) provides a non-circular confirmation of $\alpha = 0.5$. In dwarf galaxies ($v_{\text{flat}} < 47$ km/s), $b_{\text{gc}} = 2.32$: the membrane fold state dominates rotation velocity, a distinctive prediction with no analogue in standard dark matter models.

6.6 Deep-MOND g_c - $\text{Sigma}_{\text{bar}}$ independence

In the deep-MOND regime (bottom 30% by g_c), g_c is uncorrelated with baryonic surface density: $r(g_c, \text{Sigma}_{\text{bar}}) = +0.066$. In contrast, $r(g_c, \text{Sigma}_{\text{dyn}}) = +0.727$. The ratio $\text{Sigma}_{\text{bar}}/\text{Sigma}_{\text{dyn}}$ anti-correlates with g_c ($r = -0.648$): from ~ 0.7 in low- g_c galaxies to ~ 0.14 in high- g_c . The membrane responds to dynamical stress-energy, not baryonic mass content. The self-referential closure $g_c \sim \text{Sigma}_{\text{bar}}^{1/3}$ fails because its premise is empirically false. Grade: A.

6.7 Intrinsic scatter limit

Two independent optimisation pathways fail to reduce the 0.286 dex scatter: (i) $\alpha(g_c)$ tanh transition achieves 3% improvement ($\text{dAIC} = -2.4$, 4 extra parameters -- insufficient); (ii) all additional galaxy parameters achieve at most 1.7% (LOO). The scatter-minimising $\alpha(g_c)$ parameters contradict the sliding-window α in direction, confirming that local

statistical slopes and global prediction accuracy are distinct quantities. The Rotmod-derived local g_c (0.244 dex) provides a lower bound. Grade: A.

6.8 Plasticity narrative

The Yd dependence of eta is a structural consequence of the BTFR + GML system, not a "plasticity proxy". All plasticity indicators (f_{gas} , SBdisk₀, T-type) are different aspects of Σ_{dyn} variation already captured by C15. The morphological-type bridge (T-type vs S_{gal} , $\rho = +0.89$, $p = 4 \times 10^{-60}$) establishes a strong conceptual correspondence but adds no predictive power. "Plasticity" is an explanatory narrative, not a predictive variable. Grade: A (bridge) / X (S_{gal} prediction).

6.8.1 Partial first-principles derivation of the Yd exponent

The C15 dependence $g_c \sim Yd^{-0.36}$ can be partially grounded in the membrane free-energy structure. Writing the elastic fraction $s = 1 - f_{\text{gas}}$ as a Boltzmann partition over the energy barrier $DU(c) = U(\text{eps}_c; c) - U(0; c)$, the self-consistent equation $s = 1/(1 + \exp(-DU(s(Q)/T_m))$ determines $s(Q)$ for each galaxy, where $Q = \sqrt{v_{\text{flat}}^2/(a_0 h_R)}$ encodes the dynamical surface density.

Three results emerge. First, the SPARC sample-averaged elastic fraction is $\langle s \rangle \sim 0.58$, providing a physical origin for the C15 normalisation $\eta_0 = 0.584$: the mean elastic fraction of the membrane across the observed galaxy population. Second, the residual correlation between g_c and Yd after removing the $s(Q)$ dependence is $r = -0.30$ ($p = 7 \times 10^{-5}$), demonstrating that Yd is a genuinely independent variable not derivable from Q alone. The membrane state space is therefore at least two-dimensional: dynamical structure (Q) and material properties (Yd). Third, adopting $T_m = \sqrt{6}$ from the Z2 SSB critical temperature, the extended model $g_c = A s(Q) Q Yd^{\text{beta}}$ yields $\text{beta} = -0.33$, within 8% of the observed -0.36, with scatter = 0.285 dex (comparable to C15). Grade: B+ (partial derivation; full A-grade requires exact beta and independent T_m verification).

6.9 Weak Lensing RAR: KiDS-1000 and HSC-SSP Y3 (v4.7.4)

We examined the KiDS-1000 weak lensing Radial Acceleration Relation (Brouwer et al. 2021) for consistency with C15. The KiDS lensing RAR extends the rotation-curve RAR by two decades in g_{obs} into the low-acceleration regime ($g_{\text{obs}} \sim 10^{-15}$ to 10^{-12} m/s^2), using stacked Excess Surface Density (ESD) profiles of ~ 1 million isolated galaxies from the KiDS-bright sample.

Following Brouwer et al., we adopt their nominal circumgalactic gas (CGM) model ($M_{\text{hot}}/M_* = 1$, isothermal profile, truncation radius $R_{\text{acc}} = 100 \text{ kpc}$). A universal correction factor $C(g_{\text{bar}}) = \text{ESD}_{\text{hotgas}}(g_{\text{bar}}) / \text{ESD}_{\text{no-hotgas}}(g_{\text{bar}})$ is derived from the full-sample (no mass bins) ESD pair and applied to four stellar mass bins ($\log M_*/M_{\text{sun}} = [8.5, 10.3, 10.6, 10.8, 11.0]$). Per-bin g_c is then fitted using the membrane RAR.

Under the nominal CGM model, the hot-gas-corrected per-bin g_c values show a positive M_* slope (+0.138 in $\log g_c$ vs $\log M_*$), consistent in sign and order of magnitude with the C15 prediction (+0.075). The per-bin ratio $g_{c,\text{obs}}/g_{c,\text{C15}}$ ranges from 0.77 to 1.05 across all four bins, with Bin 4 ($\log M_* = 10.9$) achieving exact agreement (ratio = 1.00).

However, a systematic sensitivity analysis reveals that this result is not robust to CGM mass uncertainty. Scanning M_{hot}/M_* over the observationally permitted range [0.3, 3.0] (Tumlinson et al. 2017; Babyk et al. 2018), the formal $\Delta\text{-}\chi^2(\text{MOND} - \text{C15})$ computed with the full 60 x 60 covariance matrix reverses sign: $\Delta\text{-}\chi^2 = -10.6$ at $f_{\text{gas}} = 0.3$ (MOND preferred) versus +147.7 at $f_{\text{gas}} = 1.0$ (C15 preferred). Introducing a physically motivated M_* -dependent correction (R_{acc} proportional to $M_*^{0.4}$) further reduces the C15 preference to $\Delta\text{-}\chi^2 = +0.8$ at $f_{\text{gas}} = 1.0$, rendering the two models statistically indistinguishable.

The Brouwer et al. colour and Sersic-index bins (their Fig. 8) provide a complementary test. Splitting by colour or Sersic index, we find $g_c(\text{red})/g_c(\text{blue}) = 2.3$ to 2.8 and $g_c(\text{high-n})/g_c(\text{low-n}) = 2.4$ to 2.5, with the ordering stable across $f_{\text{gas}} = 0$ to 1.0 ($\Delta\text{-}\chi^2(\text{uniform} - \text{split}) = +127$ for colour, +67 for Sersic). This result is CGM-robust because the hot-gas correction affects both morphological bins approximately equally, preserving the g_c ratio.

However, a SPARC cross-check reveals that this ordering is dominated by the colour-magnitude relation rather than a pure morphological effect. In SPARC, the median g_c ratio between early-type ($T \leq 3$) and late-type ($T > 3$) galaxies is 0.92, with no statistically significant difference (Mann-Whitney $p = 0.35$). C15 predicts a ratio of 1.7, arising entirely from $\Sigma_{\text{dyn}} = v_{\text{flat}}^2/h_R$ differences, with zero contribution from Yd (both groups share median Yd = 0.30 in the TA3 sample). The KiDS ratio of 2.3 to 2.8 exceeds both the SPARC observation and the C15 prediction, indicating that the colour/Sersic bins primarily separate galaxies by stellar mass rather than by morphological type, with the g_c ordering reflecting the underlying M_* - Σ_{dyn} correlation.

In summary, the KiDS lensing data are consistent with both C15 and MOND at the current level of systematic uncertainty. The stellar-mass test cannot discriminate between the two models due to CGM mass uncertainty, and the colour/Sersic test, while CGM-robust, is dominated by the colour-magnitude relation rather than an independent morphological signal. Definitive tests will require future surveys combining high-precision weak lensing (Euclid, Rubin LSST) with resolved CGM measurements (eROSITA, SKA), and colour-binned analysis within narrow stellar-mass slices to isolate pure morphological

effects. Grade: B (both tests). M_* test: CGM-limited. Colour/Sersic test: CGM-robust but colour- M_* degenerate.

HSC-SSP Y3 independent lensing RAR (v4.7.4)

To provide an independent test of the lensing RAR obtained with KiDS-1000, we constructed a fully independent weak-lensing pipeline using the Hyper Suprime-Cam Subaru Strategic Program Year 3 (HSC-SSP Y3) shape catalogue as source galaxies and the Galaxy and Mass Assembly Data Release 4 (GAMA DR4) spectroscopic survey as lens galaxies. The HSC-SSP Y3 catalogue contains 35.7 million galaxies with shapes measured via the Re-Gaussianization (regauss) method, reaching an effective source number density of 19.9 arcmin^{-2} with a median i-band seeing of 0.6 arcsec. GAMA DR4 provides accurate spectroscopic redshifts ($N_Q \geq 3$) and stellar masses across three equatorial fields (G09, G12, G15), each spanning 60 deg^2 .

We selected isolated lens galaxies from GAMA DR4 via the G3C group catalogue, retaining only central galaxies ($\text{RankIsterCen} = 1$) with $0.05 < z < 0.5$ and $\log_{10}(M_*/M_{\text{sun}}) < 11.0$, the latter matching the isolation criterion of Brouwer et al. (2021). This yielded 157,338 isolated lenses across three fields: 49,272 (G09), 55,402 (G12), 52,664 (G15). The baryonic acceleration was computed per lens as $g_{\text{bar}} = G M_{\text{gal}} / R^2$, treating each lens as a baryonic point mass (Brouwer et al. 2021, Eq. 2), with pairs binned in 15 logarithmic g_{bar} bins spanning 10^{-15} to $5 \times 10^{-12} \text{ m s}^{-2}$. A total of 503 million lens-source pairs were accumulated.

Systematic corrections were applied as follows. The HSC regauss shear responsivity $R = 1 - \langle e_{\text{rms}}^2 \rangle$ was computed per field ($R \sim 0.86$). The multiplicative shear bias for HSC Y3 regauss is controlled at $|m| < 10^{-2}$ through image-simulation calibration (Li et al. 2022; Mandelbaum et al. 2018), contributing $<1\%$ ESD uncertainty. The observed acceleration was obtained via the SIS approximation $g_{\text{obs}} = 4G \Delta\text{-Sigma}$ (Brouwer et al. 2021, Eq. 7). The cross component ESD_x is consistent with zero across all bins, confirming the absence of significant additive systematics. Statistical uncertainties were estimated using a 24-patch spatial jackknife (8 per field) yielding the full 15×15 covariance matrix.

A critical systematic-robustness test is the consistency of results across independent survey fields. Fitting the interpolation $g_{\text{obs}} = g_{\text{bar}} / (1 - \exp(-\sqrt{g_{\text{bar}}/g_c}))$ independently in each field yields $g_c = 2.91 \pm 0.19 a_0$ (G09), $2.64 \pm 0.16 a_0$ (G12), $2.64 \pm 0.23 a_0$ (G15). The inter-field consistency $\chi^2 = 1.37$ (2 d.o.f., $p = 0.50$) confirms that all three fields are statistically consistent at the 1-sigma level, with no evidence for field-dependent systematics.

Combining all three fields yields $g_c = 2.73 \pm 0.11 a_0$ for the full isolated lens sample. Comparing C15 (g_c free) to MOND ($g_c = a_0$ fixed) using the full jackknife covariance matrix yields $\chi^2_{\text{MOND}} = 539.3$ versus $\chi^2_{\text{C15}} = 65.0$, giving $\Delta\text{-}\chi^2 = +474$ and $\Delta\text{-AIC} = +472$, corresponding to a ~ 22 -sigma preference for C15 over MOND. The lensing RAR of isolated galaxies at Mpc scales is incompatible with a universal acceleration scale $g_c = a_0$ as predicted by MOND.

The g_c - M_* relation provides a further discriminant. Fitting g_c in four stellar-mass bins between $\log_{10} M_* = 8.5$ and 11.0, we find a monotonic increase from $g_c = 2.00 \pm 0.27 a_0$ (lowest M_*) to $3.41 \pm 0.21 a_0$ (highest M_*), with a log-log slope of $+0.166 \pm 0.041$. MOND predicts zero slope and is rejected at 4.0 sigma ($p < 0.001$). C15, which predicts a positive slope of $+0.075$ from the $\text{Sigma}_{\text{dyn}}$ dependence, is consistent at the 2.2-sigma level ($p = 0.029$). The observed slope being approximately twice the $\text{Sigma}_{\text{dyn}}$ prediction suggests that the $\text{Sigma}_{\text{dyn}}$ dependence at Mpc lensing scales may be steeper than at kpc rotation-curve scales, potentially reflecting the transition from baryon-dominated to halo-dominated regimes. This is a quantitative prediction for future refinement of the membrane framework.

In summary, the HSC-SSP Y3 lensing RAR independently confirms the C15 framework at high significance: (i) $g_c = 2.73 \pm 0.11 a_0$ is inconsistent with the MOND universal value a_0 at >15 sigma; (ii) the g_c - M_* slope rejects MOND at 4 sigma while remaining consistent with C15's predicted positive slope; (iii) all three GAMA fields yield consistent g_c values ($p = 0.50$); (iv) combined with the KiDS-1000 analysis, the lensing RAR evidence upgrades from B grade (KiDS alone, CGM-limited) to A grade (independent HSC confirmation with systematic verification). The observed factor ~ 2 excess of the slope over the C15 prediction is noted as a quantitative tension requiring theoretical investigation, but does not affect the qualitative conclusion that g_c varies with galaxy properties as predicted by membrane cosmology and contrary to MOND.

To quantify the 2-halo contribution to the slope excess, we re-fitted g_c in each M_* bin using only the inner radial range ($R < 200 \text{ kpc}$, where one-halo signal dominates) and compared with the full-range fit. The empirical two-halo slope contribution is $\Delta\text{-slope} = +0.034 \pm 0.064$ (0.5 sigma, suggestive but not individually significant). Combined with an analogous geometric contribution from light-propagation rewiring ($+0.017$), the total predicted slope becomes C15 ($+0.075$) + geometric ($+0.017$) + two-halo ($+0.034$) = $+0.126$, reducing the residual tension with the observed $+0.166 \pm 0.041$ from 2.2 sigma to ~ 1 sigma. Full resolution requires Euclid Wide survey statistics, which should reach 2-3 sigma detection of the two-halo slope contribution.

7. Conclusions

We have presented a membrane cosmology framework with the following established results:

1. Geometric mean law (A-grade): $g_c = \eta \sqrt{a_0 G \text{Sigma}_0}$, $\alpha = 0.5$, $\text{dAIC} = -130$ vs MOND, 175 SPARC galaxies. Jackknife-verified (96.5%). Origin: deep-MOND algebraic consequence ($\alpha = 0.471 \pm 0.036$, $p(0.5) = 0.43$).

- 2. MOND from first principles (A-grade):** Simple interpolation $\mu(x) = x/(1+x)$ derived from elastic membrane potential equilibrium. Not an empirical ansatz.
- 3. C15 final form (A-grade):** $g_c = 0.584 Yd^{-0.361} \text{sqrt}(a_0 v_{\text{flat}}^2/h_R)$. $R^2 = 0.607$, scatter = 0.286 dex. MOND rejected at $p = 1.66 \times 10^{-53}$.
- 4. C15 is minimal sufficient statistic (A-grade, v4.7):** Six S_{gal} models all rejected. All parameter additions yield LOO < 2%. 94% of intrinsic variance irreducible. Scatter = membrane fold diversity.
- 5. kappa = 0: membrane rigidity zero (A-grade, v4.7):** $\kappa \sim 10^{-40} \text{kpc}^2$. $L \sim U(\epsilon; c)$. Each radius obeys $\mu(x)$ independently. Four free parameters eliminated.
- 6. C14 pattern is C15 bias projection (A-grade, v4.7):** Ratio $g_{c,\text{local}}/g_{c,\text{global}} = 1.035$. RMS = 0.090 dex. C14 dynamical formulation retracted.
- 7. F_{conf} derivation (A-grade):** $-c \ln(1-\epsilon)$ from forest + cavity MF effective theory (Lemmas 1-4).
- 8. Weak lensing Q > 1 in 9/9 clusters (A-grade):** Wilcoxon $p = 0.0020$. cl1: membrane dAIC = -4.4 vs NFW. Level B+.
- 9. Deep-MOND: g_c independent of Sigma_{bar} (A-grade):** $r(g_c, \text{Sigma}_{\text{bar}}) = 0.07$. Membrane responds to $\text{Sigma}_{\text{dyn}}$ ($r = 0.73$), not baryonic mass.
- 10. BTFR slope is Simpson's paradox artifact (A-grade):** Within-bin slope = 0.54 +/- 0.07 confirms $\alpha = 0.5$ non-circularly. Dwarf $b_{gc} = 2.32$: membrane dominates rotation.
- 11. Dwarf regime verified (B+, v4.7):** 8 LITTLE THINGS dwarfs: scatter = 0.232 dex, KS $p = 0.67$. C15 uniform across $v_{\text{flat}} = 15\text{-}300 \text{ km/s}$.
- 12. Morphological-type bridge (A-grade, v4.7):** T-type vs S_{gal} : $\rho = +0.89$, $p = 4 \times 10^{-60}$. Conceptual correspondence; no additional predictive power.
- 13. Weak lensing RAR (A grade, v4.7.4):** KiDS-1000 (Brouwer et al. 2021) constrains C15 at Mpc scales but is CGM-limited (B grade). Independent HSC-SSP Y3 + GAMA DR4 analysis (157,338 isolated lenses, 503M pairs, 3 fields) yields $g_c = 2.73 \pm 0.11 a_0$, Delta-AIC = +472 favouring C15 over MOND. g_c - M_* slope (+0.166 +/- 0.041) rejects MOND at 4 sigma. Inter-field consistency ($\chi^2 = 1.37$, $p = 0.50$) confirms systematic robustness. Combined with KiDS, the lensing RAR evidence upgrades from B to A grade via independent HSC confirmation. The factor ~2 excess of the observed slope over the C15 prediction is noted as a quantitative tension reduced from 2.2 sigma to ~1.0 sigma by empirical two-halo (+0.034) and geometric (+0.017) contributions (Sections 6.9, 6.8.1).

Retracted in v4.7:

Condition 14 dynamical formulation $g_{c,\text{eff}}(r)$; $\kappa \rightarrow A, B, \tau$ inverse estimation; Euler-Lagrange derivation pathway; gradient term $\kappa(d\epsilon/dr)^2$; S_{gal} predictive significance.

Priority future work:

(1) BIG-SPARC external verification (~4000 galaxies). (2) Non-SPARC LITTLE THINGS galaxies (Iorio et al. 2017). (3) $\epsilon_0 = 0.584$ theoretical derivation. (4) Pressure-supported systems (dSph, sqrt(3) hypothesis). (5) Rubin LSST / Euclid weak lensing.

Acknowledgments

This research is based on observations made with ESO Telescopes at the La Silla Paranal Observatory under programme IDs 177.A-3016, 177.A-3017, 177.A-3018, and 179.A-2004, and on data products produced by the KiDS consortium. The KiDS production team acknowledges support from: Deutsche Forschungsgemeinschaft; ERC; NOVA and NWO-M grants; Target; the University of Padova; and the University Federico II (Naples).

The Hyper Suprime-Cam (HSC) collaboration includes the astronomical communities of Japan and Taiwan, and Princeton University. The HSC instrumentation and software were developed by the National Astronomical Observatory of Japan (NAOJ), the Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU), the University of Tokyo, KEK, ASIAA, and Princeton University. Funding was contributed by the FIRST program from the Japanese Cabinet Office, MEXT, JSPS, JST, the Toray Science Foundation, NAOJ, Kavli IPMU, KEK, ASIAA, and Princeton University. GAMA is a joint European-Australasian project based around a spectroscopic campaign using the Anglo-Australian Telescope, funded by STFC (UK), ARC (Australia), AAO, and participating institutions.

References

- Babyk, Iu. V., et al. 2018, ApJ, 857, 32
 Brout, D., et al. 2022, ApJ, 938, 110
 Brouwer, M. M., et al. 2021, A&A, 650, A113

Dalal, R., et al. 2023, Phys. Rev. D, 108, 123519
Hunter, D. A., et al. 2012, AJ, 144, 134
Iorio, G., et al. 2017, MNRAS, 466, 4159
Julio, M. P., et al. 2025, A&A, 703, A106
Lelli, F., McGaugh, S. S., & Schombert, J. M. 2016, AJ, 152, 157
Li, X., et al. 2022, PASJ, 74, 421
Maartens, R. & Koyama, K. 2010, Living Rev. Rel., 13, 5
Mandelbaum, R., et al. 2018, MNRAS, 481, 3170
McGaugh, S. S. & Schombert, J. M. 2014, AJ, 148, 77
Meidt, S. E., et al. 2014, ApJ, 788, 144
Milgrom, M. 1983, ApJ, 270, 365
Miyaoaka, K., et al. 2019, PASJ, 71, 62
More, S., et al. 2023, Phys. Rev. D, 108, 123520
Oh, S.-H., et al. 2015, AJ, 149, 180
Randall, L. & Sundrum, R. 1999, Phys. Rev. Lett., 83, 3370
Rau, M. M., et al. 2023, MNRAS, 524, 5109
Rubin, V. C. & Ford, W. K. 1980, ApJ, 238, 471
Schombert, J. M., McGaugh, S. S., & Lelli, F. 2019, MNRAS, 483, 1496
Stone, C., et al. 2021, ApJS, 256, 33
Tully, R. B. & Fisher, J. R. 1977, A&A, 54, 661
Tumlinson, J., et al. 2017, ARA&A, 55, 389
Verlinde, E. P. 2017, SciPost Phys., 2, 016